PageRank

Ashley Montanaro
ashley@cs.bris.ac.uk

Department of Computer Science, University of Bristol
Bristol, UK

12 November 2013

Motivation

- The World-Wide Web was invented by Tim Berners-Lee circa 1991.

Motivation

- The World-Wide Web was invented by Tim Berners-Lee circa 1991.
- By the late 1990s, the amount of content on the Web had exploded, but searching this content was becoming an increasingly pressing problem.

Motivation

- The World-Wide Web was invented by Tim Berners-Lee circa 1991.
- By the late 1990s, the amount of content on the Web had exploded, but searching this content was becoming an increasingly pressing problem.
- Human-generated indices (e.g. Yahoo!) could not keep up, and automatically generated indices (e.g. AltaVista) were sometimes of low quality.

Motivation

- The World-Wide Web was invented by Tim Berners-Lee circa 1991.
- By the late 1990s, the amount of content on the Web had exploded, but searching this content was becoming an increasingly pressing problem.
- Human-generated indices (e.g. Yahoo!) could not keep up, and automatically generated indices (e.g. AltaVista) were sometimes of low quality.
- The PageRank algorithm essentially solved the web search problem and was the basis for Google's success.

Motivation

When a search query is received, satisfying that query can be thought of as a three-stage process:

Motivation

When a search query is received, satisfying that query can be thought of as a three-stage process:

1. Determine which pages can be returned (e.g. which ones contain the query string);

Motivation

When a search query is received, satisfying that query can be thought of as a three-stage process:

1. Determine which pages can be returned (e.g. which ones contain the query string);
2. Rank these pages in order of importance;

Motivation

When a search query is received, satisfying that query can be thought of as a three-stage process:

1. Determine which pages can be returned (e.g. which ones contain the query string);
2. Rank these pages in order of importance;
3. Format the ranked list and display it to the user.

Motivation

When a search query is received, satisfying that query can be thought of as a three-stage process:

1. Determine which pages can be returned (e.g. which ones contain the query string);
2. Rank these pages in order of importance;
3. Format the ranked list and display it to the user.

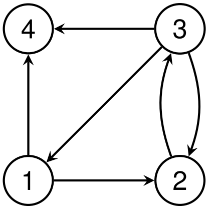
PageRank solves the second stage above. The others are also interesting algorithmic challenges (and you will hear more about the first later in this course).

Motivation

- Given a set of web pages with links between them, we would like to rank the pages in order of importance.
- We can model this as a graph problem where web pages are vertices and links are edges.
- We are given a directed, unweighted graph G, and would like to associate a real number $PR(v)$ with each vertex v which represents the importance of v.

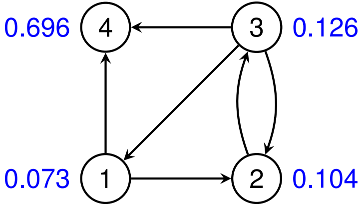
Motivation

- Given a set of web pages with links between them, we would like to rank the pages in order of importance.
- We can model this as a graph problem where web pages are vertices and links are edges.
- We are given a directed, unweighted graph G, and would like to associate a real number $PR(v)$ with each vertex v which represents the importance of v.



Motivation

- Given a set of web pages with links between them, we would like to rank the pages in order of importance.
- We can model this as a graph problem where web pages are vertices and links are edges.
- We are given a directed, unweighted graph G, and would like to associate a real number $PR(v)$ with each vertex v which represents the importance of v.



"Importance"

- What does this even mean?

"Importance"

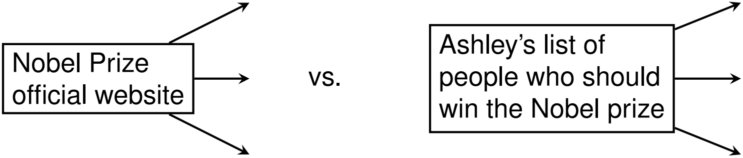
- What does this even mean?
- Intuitively, we might think that a web page is important if many other pages link to it; each link provides a "recommendation" that the page is worth visiting.

“Importance”

- What does this even mean?
- Intuitively, we might think that a web page is important if many other pages link to it; each link provides a “recommendation” that the page is worth visiting.
- However, not all links are created equal…

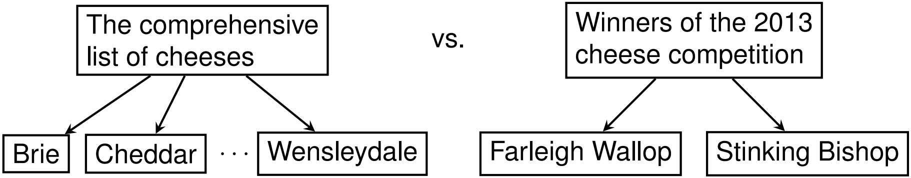
“Importance”

- What does this even mean?
- Intuitively, we might think that a web page is important if many other pages link to it; each link provides a “recommendation” that the page is worth visiting.
- However, not all links are created equal. . .
- A link from an important web page is more significant than a link from an unimportant web page, so should be “worth” more, e.g.



“Importance”

Also, being linked from a page which has many outgoing links is less significant than being linked from a page with only a few outgoing links, e.g.



Intuitively, the importance of a page is “diluted” by having too many outgoing links.

Simplified PageRank

The following quantity encapsulates these two ideas:

Definition (Simplified PageRank)

The simplified PageRank of a vertex v is the real number $R(v)$ satisfying the equation

$$R(v) = \sum_{u \in B(v)} \frac{R(u)}{\deg(u)}.$$

In the above definition:

- $B(v)$ is the set of backlinks from v, i.e. the set of vertices u such that there is an edge $u \to v$.
- $\deg(u)$ is the degree of u, i.e. the number of outgoing edges.

Simplified PageRank

The following quantity encapsulates these two ideas:

Definition (Simplified PageRank)

The simplified PageRank of a vertex v is the real number $R(v)$ satisfying the equation

$$R(v) = \sum_{u \in B(v)} \frac{R(u)}{\deg(u)}.$$

In the above definition:

- $B(v)$ is the set of backlinks from v, i.e. the set of vertices u such that there is an edge $u \to v$.
- $\deg(u)$ is the degree of u, i.e. the number of outgoing edges.

(Does such a function R actually exist, and is it unique? See later...)

The random surfer model

We now discuss an alternative approach one could follow for ranking pages on the Web, which is known as the random surfer model. This will turn out to be basically equivalent.

The random surfer model

We now discuss an alternative approach one could follow for ranking pages on the Web, which is known as the random surfer model. This will turn out to be basically equivalent.

- Imagine a user browsing the Web. The user starts at an arbitrary page, and follows links at random forever, or until he gets stuck (i.e. comes to a page with no outgoing links).

The random surfer model

We now discuss an alternative approach one could follow for ranking pages on the Web, which is known as the random surfer model. This will turn out to be basically equivalent.

- Imagine a user browsing the Web. The user starts at an arbitrary page, and follows links at random forever, or until he gets stuck (i.e. comes to a page with no outgoing links).
- This can be modelled as a random walk on the web graph. At each step, the user picks an outgoing edge at random. For vertices u with no outgoing edges, we can add a single edge $u \to u$, so the user stays at the current page (or just ignore all such vertices).

The random surfer model

We now discuss an alternative approach one could follow for ranking pages on the Web, which is known as the random surfer model. This will turn out to be basically equivalent.

- Imagine a user browsing the Web. The user starts at an arbitrary page, and follows links at random forever, or until he gets stuck (i.e. comes to a page with no outgoing links).
- This can be modelled as a random walk on the web graph. At each step, the user picks an outgoing edge at random. For vertices u with no outgoing edges, we can add a single edge $u \to u$, so the user stays at the current page (or just ignore all such vertices).
- We then define the (variant) simplified PageRank $R'(v)$ as the probability of being at vertex v after k steps, starting at an arbitrary vertex, as $k \to \infty$.

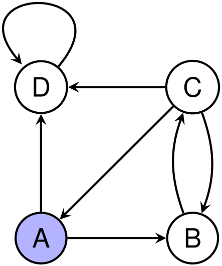
The random surfer model

We now discuss an alternative approach one could follow for ranking pages on the Web, which is known as the random surfer model. This will turn out to be basically equivalent.

- Imagine a user browsing the Web. The user starts at an arbitrary page, and follows links at random forever, or until he gets stuck (i.e. comes to a page with no outgoing links).
- This can be modelled as a random walk on the web graph. At each step, the user picks an outgoing edge at random. For vertices u with no outgoing edges, we can add a single edge $u \to u$, so the user stays at the current page (or just ignore all such vertices).
- We then define the (variant) simplified PageRank $R'(v)$ as the probability of being at vertex v after k steps, starting at an arbitrary vertex, as $k \to \infty$.

(Is this well-defined? See later…)

Example



Connecting the two notions of simplified PageRank

These two ideas for how to define PageRank are essentially equivalent, which we can show using ideas from linear algebra.

Connecting the two notions of simplified PageRank

These two ideas for how to define PageRank are essentially equivalent, which we can show using ideas from linear algebra.

- Let A be the matrix whose rows and columns are indexed by vertices, and

$$A_{uv} = \begin{cases} \frac{1}{\deg(u)} & \text{if there is an edge } u \to v \\ 0 & \text{otherwise.} \end{cases}$$

Connecting the two notions of simplified PageRank

These two ideas for how to define PageRank are essentially equivalent, which we can show using ideas from linear algebra.

- Let A be the matrix whose rows and columns are indexed by vertices, and

$$A_{uv} = \begin{cases} \frac{1}{\deg(u)} & \text{if there is an edge } u \to v \\ 0 & \text{otherwise.} \end{cases}$$

- Assume that the user starts at vertex w. If the probability that the user is at vertex u after k steps is $p_u^{(k)}$, then the probability that the user is at vertex v after $k+1$ steps is precisely

$$\sum_u \Pr[\text{at vertex } u \text{ after } k \text{ steps}] \times \Pr[\text{move from } u \text{ to } v] = \sum_u p_u^{(k)} A_{uv}.$$

Connecting the two notions of simplified PageRank

These two ideas for how to define PageRank are essentially equivalent, which we can show using ideas from linear algebra.

- Let A be the matrix whose rows and columns are indexed by vertices, and

$$A_{uv} = \begin{cases} \frac{1}{\deg(u)} & \text{if there is an edge } u \to v \\ 0 & \text{otherwise.} \end{cases}$$

- Assume that the user starts at vertex w. If the probability that the user is at vertex u after k steps is $p_u^{(k)}$, then the probability that the user is at vertex v after $k+1$ steps is precisely

$$\sum_u \Pr[\text{at vertex } u \text{ after } k \text{ steps}] \times \Pr[\text{move from } u \text{ to } v] = \sum_u p_u^{(k)} A_{uv}.$$

- In vector form, we can write

$$p^{(k+1)} = p^{(k)} A.$$

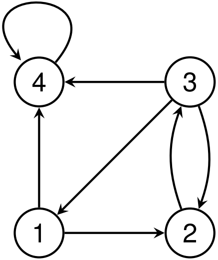
Connecting the two notions of simplified PageRank

- For any vertex v, let e_v denote the vector whose components are indexed by vertices, and which is 1 at position v, and 0 elsewhere. Then $p^{(0)} = e_w$, so
$$p^{(k)} = e_w A^k.$$

Connecting the two notions of simplified PageRank

- For any vertex v, let e_v denote the vector whose components are indexed by vertices, and which is 1 at position v, and 0 elsewhere. Then $p^{(0)} = e_w$, so
$$p^{(k)} = e_w A^k.$$

For example:

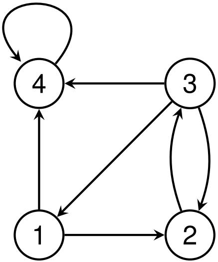


$$A = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Connecting the two notions of simplified PageRank

- For any vertex v, let e_v denote the vector whose components are indexed by vertices, and which is 1 at position v, and 0 elsewhere. Then $p^{(0)} = e_w$, so
$$p^{(k)} = e_w A^k.$$

For example:



$$A = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

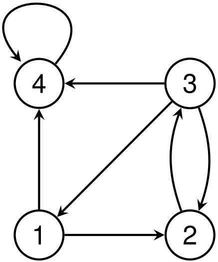
$$p^{(0)} = (1, 0, 0, 0)$$

Connecting the two notions of simplified PageRank

- For any vertex v, let e_v denote the vector whose components are indexed by vertices, and which is 1 at position v, and 0 elsewhere. Then $p^{(0)} = e_w$, so

$$p^{(k)} = e_w A^k.$$

For example:



$$A = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

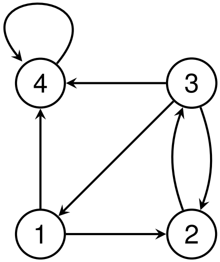
$p^{(0)} = (1, 0, 0, 0)$ $\quad$ $p^{(1)} = (0, 1/2, 0, 1/2)$

Connecting the two notions of simplified PageRank

- For any vertex v, let e_v denote the vector whose components are indexed by vertices, and which is 1 at position v, and 0 elsewhere. Then $p^{(0)} = e_w$, so

$$p^{(k)} = e_w A^k.$$

For example:



$$A = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

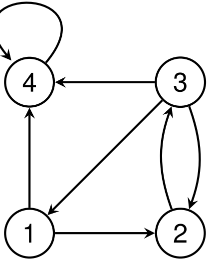
$p^{(0)} = (1, 0, 0, 0)$ $\quad p^{(1)} = (0, 1/2, 0, 1/2)$ $\quad p^{(2)} = (0, 0, 1/2, 1/2)$

Connecting the two notions of simplified PageRank

- For any vertex v, let e_v denote the vector whose components are indexed by vertices, and which is 1 at position v, and 0 elsewhere. Then $p^{(0)} = e_w$, so

$$p^{(k)} = e_w A^k.$$

For example:



$$A = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$p^{(0)} = (1, 0, 0, 0)$ $\quad$ $p^{(1)} = (0, 1/2, 0, 1/2)$ $\quad$ $p^{(2)} = (0, 0, 1/2, 1/2)$

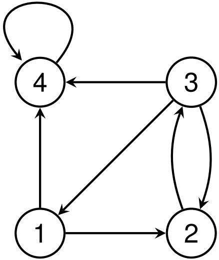
$p^{(3)} = (1/6, 1/6, 0, 2/3)$

Connecting the two notions of simplified PageRank

- For any vertex v, let e_v denote the vector whose components are indexed by vertices, and which is 1 at position v, and 0 elsewhere. Then $p^{(0)} = e_w$, so

$$p^{(k)} = e_w A^k.$$

For example:



$$A = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$p^{(0)} = (1, 0, 0, 0)$ $p^{(1)} = (0, 1/2, 0, 1/2)$ $p^{(2)} = (0, 0, 1/2, 1/2)$

$p^{(3)} = (1/6, 1/6, 0, 2/3)$ $p^{(4)} = (0, 1/12, 1/6, 3/4)$

Connecting the two notions of simplified PageRank

- By our second definition of simplified PageRank R', if there exists a vector π such that $\lim_{k \to \infty} e_w A^k = \pi$ for any starting vertex w, then $R'(v) = \pi_v$.

Connecting the two notions of simplified PageRank

- By our second definition of simplified PageRank R', if there exists a vector π such that $\lim_{k \to \infty} e_w A^k = \pi$ for any starting vertex w, then $R'(v) = \pi_v$.
- If such a probability distribution π exists, we must have $\pi A = \pi$, so π is an eigenvector of the matrix A with eigenvalue 1. π is called a stationary distribution of the random walk.

Connecting the two notions of simplified PageRank

- By our second definition of simplified PageRank R', if there exists a vector π such that $\lim_{k\to\infty} e_w A^k = \pi$ for any starting vertex w, then $R'(v) = \pi_v$.
- If such a probability distribution π exists, we must have $\pi A = \pi$, so π is an eigenvector of the matrix A with eigenvalue 1. π is called a stationary distribution of the random walk.
- If such a distribution π exists, R' satisfies the first definition of simplified PageRank too:

$$\pi A = \pi \Leftrightarrow \sum_{u \in B(v)} \frac{R'(u)}{\deg(u)} = R'(v) \text{ for all } v.$$

Connecting the two notions of simplified PageRank

- By our second definition of simplified PageRank R', if there exists a vector π such that $\lim_{k\to\infty} e_w A^k = \pi$ for any starting vertex w, then $R'(v) = \pi_v$.
- If such a probability distribution π exists, we must have $\pi A = \pi$, so π is an eigenvector of the matrix A with eigenvalue 1. π is called a stationary distribution of the random walk.
- If such a distribution π exists, R' satisfies the first definition of simplified PageRank too:

$$\pi A = \pi \Leftrightarrow \sum_{u \in B(v)} \frac{R'(u)}{\deg(u)} = R'(v) \text{ for all } v.$$

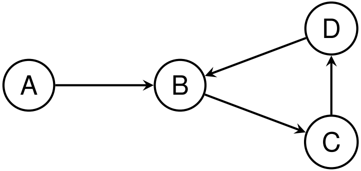
- For the graph on the previous slide, $\pi = (0, 0, 0, 1)$ works.

Problems with the simplified PageRank

- There are two problems with this simplified model of PageRank. First, such a stationary distribution π might not exist (or be unique).

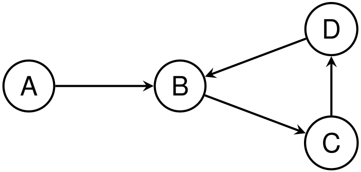
Problems with the simplified PageRank

- There are two problems with this simplified model of PageRank. First, such a stationary distribution π might not exist (or be unique).
- Second, there is a conceptual problem highlighted by the random surfer model: if there is a page which links into some subset of webpages which don't connect to anything else, the surfer will never leave this subset of webpages.



Problems with the simplified PageRank

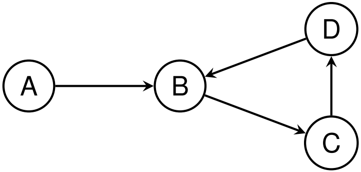
- There are two problems with this simplified model of PageRank. First, such a stationary distribution π might not exist (or be unique).
- Second, there is a conceptual problem highlighted by the random surfer model: if there is a page which links into some subset of webpages which don't connect to anything else, the surfer will never leave this subset of webpages.



- For example, taking the above graph the simplified PageRank of A will be 0.

Problems with the simplified PageRank

- There are two problems with this simplified model of PageRank. First, such a stationary distribution π might not exist (or be unique).
- Second, there is a conceptual problem highlighted by the random surfer model: if there is a page which links into some subset of webpages which don't connect to anything else, the surfer will never leave this subset of webpages.



- For example, taking the above graph the simplified PageRank of A will be 0.
- We can fix this by including some probability p for the surfer to get bored and go to a random web page.

PageRank

Definition (PageRank)

The PageRank of a vertex v is the real number $PR(v)$ satisfying the equation

$$PR(v) = \frac{p}{N} + (1-p) \sum_{u \in B(v)} \frac{PR(u)}{\deg(u)}.$$

Here N is the total number of vertices and p is some constant between 0 and 1 giving the “probability of boredom”. $p \approx 0.15$ is often used.

PageRank

Definition (PageRank)

The PageRank of a vertex v is the real number $PR(v)$ satisfying the equation

$$PR(v) = \frac{p}{N} + (1-p) \sum_{u \in B(v)} \frac{PR(u)}{\deg(u)}.$$

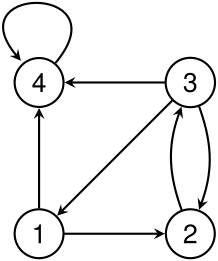
Here N is the total number of vertices and p is some constant between 0 and 1 giving the "probability of boredom". $p \approx 0.15$ is often used.

- This is equivalent to modifying A to be of the form

$$A'_{uv} = \begin{cases} \frac{p}{N} + \frac{1-p}{\deg(u)} & \text{if there is an edge } u \to v \\ \frac{p}{N} & \text{otherwise.} \end{cases}$$

Example

Using the same graph as before, and taking $p = 0.15$:

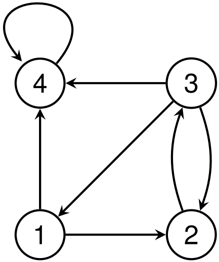


$$A = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A' \approx \begin{pmatrix} 0.0375 & 0.463 & 0.0375 & 0.463 \\ 0.0375 & 0.0375 & 0.888 & 0.0375 \\ 0.321 & 0.321 & 0.0375 & 0.321 \\ 0.0375 & 0.0375 & 0.0375 & 0.888 \end{pmatrix}$$

Example

Using the same graph as before, and taking $p = 0.15$:



$$A = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A' \approx \begin{pmatrix} 0.0375 & 0.463 & 0.0375 & 0.463 \\ 0.0375 & 0.0375 & 0.888 & 0.0375 \\ 0.321 & 0.321 & 0.0375 & 0.321 \\ 0.0375 & 0.0375 & 0.0375 & 0.888 \end{pmatrix}$$

- It turns out that $\pi \approx (0.073, 0.104, 0.126, 0.696)$ satisfies $\pi A' = A'$.
- So $PR(1) \approx 0.073$, $PR(2) \approx 0.104$ etc.

PageRank

We have seen that PR, if it exists, corresponds to an eigenvector of A' with eigenvalue 1.

Theorem (19th century)

Any matrix describing a random walk on a graph has all eigenvalues in the range $[-1, 1]$, and has an eigenvector with eigenvalue 1. Further, the entries of this eigenvector are non-negative.

PageRank

We have seen that *PR*, if it exists, corresponds to an eigenvector of A' with eigenvalue 1.

Theorem (19th century)

Any matrix describing a random walk on a graph has all eigenvalues in the range $[-1, 1]$, and has an eigenvector with eigenvalue 1. Further, the entries of this eigenvector are non-negative.

So *PR* exists. To show it is the unique such eigenvector requires a bit more work. In fact, the following stronger result is known.

Theorem (Haveliwala and Kamvar '03)

The second-largest eigenvalue λ_2 of A' satisfies $|\lambda_2| \leq 1 - p$.

Computing the PageRank

- The random surfer model allows us to compute PageRank efficiently.

Computing the PageRank

- The random surfer model allows us to compute PageRank efficiently.
- As PR is the stationary distribution of the random walk, we simply apply A' repeatedly, until the resulting vector does not change much. We must then have approximated the stationary distribution of A'.

Computing the PageRank

- The random surfer model allows us to compute PageRank efficiently.
- As *PR* is the stationary distribution of the random walk, we simply apply A' repeatedly, until the resulting vector does not change much. We must then have approximated the stationary distribution of A'.

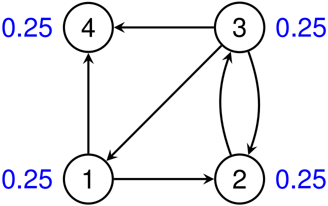
PageRank()

1. for all i, $v_i^{(0)} \leftarrow 1/N$
2. $k \leftarrow 0$
3. repeat forever
4. $\quad v^{(k+1)} \leftarrow v^{(k)} A'$
5. $\quad$ if $\sum_i |v_i^{(k+1)} - v_i^{(k)}| \leq \epsilon$
6. $\quad\quad$ return $v^{(k+1)}$
7. $\quad k \leftarrow k + 1$

Here ϵ is an arbitrary small parameter specifying the desired accuracy.

Example

At the start of the algorithm:

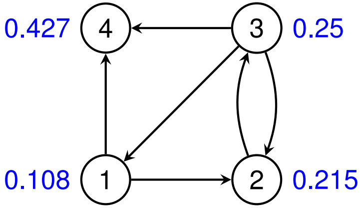


$$A' \approx \begin{pmatrix} 0.0375 & 0.463 & 0.0375 & 0.463 \\ 0.0375 & 0.0375 & 0.888 & 0.0375 \\ 0.321 & 0.321 & 0.0375 & 0.321 \\ 0.0375 & 0.0375 & 0.0375 & 0.888 \end{pmatrix}$$

- Blue labels: the updating PageRanks of the vertices.

Example

After one iteration:

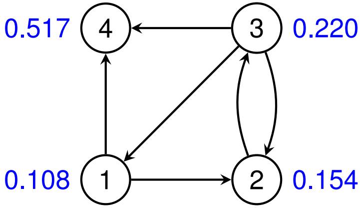


$$A' \approx \begin{pmatrix} 0.0375 & 0.463 & 0.0375 & 0.463 \\ 0.0375 & 0.0375 & 0.888 & 0.0375 \\ 0.321 & 0.321 & 0.0375 & 0.321 \\ 0.0375 & 0.0375 & 0.0375 & 0.888 \end{pmatrix}$$

- Blue labels: the updating PageRanks of the vertices.

Example

After two iterations:

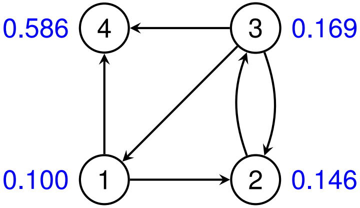


$$A' \approx \begin{pmatrix} 0.0375 & 0.463 & 0.0375 & 0.463 \\ 0.0375 & 0.0375 & 0.888 & 0.0375 \\ 0.321 & 0.321 & 0.0375 & 0.321 \\ 0.0375 & 0.0375 & 0.0375 & 0.888 \end{pmatrix}$$

- Blue labels: the updating PageRanks of the vertices.

Example

After three iterations:



$$A' \approx \begin{pmatrix} 0.0375 & 0.463 & 0.0375 & 0.463 \\ 0.0375 & 0.0375 & 0.888 & 0.0375 \\ 0.321 & 0.321 & 0.0375 & 0.321 \\ 0.0375 & 0.0375 & 0.0375 & 0.888 \end{pmatrix}$$

- Blue labels: the updating PageRanks of the vertices.

Runtime

- The result that $|\lambda_2| \leq 1 - p$, i.e. is a constant strictly less than 1, turns out to imply that it suffices to repeat the above loop $O(\log N)$ times to achieve a value of ϵ which is a small constant (e.g. 0.001).

Runtime

- The result that $|\lambda_2| \leq 1 - p$, i.e. is a constant strictly less than 1, turns out to imply that it suffices to repeat the above loop $O(\log N)$ times to achieve a value of ϵ which is a small constant (e.g. 0.001).
- The algorithm is very efficient in practice; in their 1998 paper, Brin and Page report that the PageRank of all pages in a 322 million page database can be approximately computed in 52 iterations.

Runtime

- The result that $|\lambda_2| \leq 1 - p$, i.e. is a constant strictly less than 1, turns out to imply that it suffices to repeat the above loop $O(\log N)$ times to achieve a value of ϵ which is a small constant (e.g. 0.001).
- The algorithm is very efficient in practice; in their 1998 paper, Brin and Page report that the PageRank of all pages in a 322 million page database can be approximately computed in 52 iterations.
- The multiplication by A' can also be performed very efficiently, as the "web graph" is sparse. One iteration takes time $O(N + L)$, where L is the number of links on the web. (Multiplication by a general matrix would take time $\Theta(N^2)$.)

Does this make sense?

Is PageRank actually a sensible way to rank pages?

- Ultimately, the metric that measures PageRank's success is its usefulness to its users.
- Nowadays Google uses a number of other methods to rank pages (most of them secret), including AI / machine learning techniques. As well as improving search quality in general, this is helpful to avoid spam and other undesirable pages.
- Companies like Google perform extensive user testing and validation to determine whether their algorithms actually work.

Summary

- The PageRank algorithm has enabled web search to keep pace with the hugely increasing quantities of data on the Internet. It also has a number of other applications (e.g. ranking academic research using the graph of citations).

Summary

- The PageRank algorithm has enabled web search to keep pace with the hugely increasing quantities of data on the Internet. It also has a number of other applications (e.g. ranking academic research using the graph of citations).
- Developing an index of importance of web pages can be done quite accurately by modelling humans as clicking on links at random, occasionally getting bored and going to a completely random page.

Summary

- The PageRank algorithm has enabled web search to keep pace with the hugely increasing quantities of data on the Internet. It also has a number of other applications (e.g. ranking academic research using the graph of citations).
- Developing an index of importance of web pages can be done quite accurately by modelling humans as clicking on links at random, occasionally getting bored and going to a completely random page.
- Google, a $300 billion company, ultimately stems from an efficient algorithm and some Victorian-era linear algebra.

Further reading

- "The PageRank Citation Ranking: Bringing Order to the Web"
  L. Page and S. Brin and R. Motwani and T. Winograd
  http://ilpubs.stanford.edu:8090/422/

- "The Anatomy of a Large-Scale Hypertextual Web Search Engine"
  S. Brin and L. Page
  http://ilpubs.stanford.edu:8090/361/

- A short lecture course on PageRank and other Google technology:
  http://michaelnielsen.org/blog/
  lecture-course-the-google-technology-stack/

Historical notes

- PageRank was developed by graduate students Sergey Brin and Larry Page, who went on to drop out of their PhDs and found Google.
- Similar ideas had been developed by some other people previously and concurrently (e.g. Robin Li, Jon Kleinberg).
- Although PageRank is a method for ranking pages, it was in fact named after Larry Page.




Pics: engadget.com, wired.com, cnn.com, acm.org