

Quantum Computing

Ashley Montanaro

ashley@cs.bris.ac.uk

Department of Computer Science, University of Bristol
Bristol, UK

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Google tries to save the world: Internet giant explains how its move into quantum computing could solve global warming

- Google's D-Wave computer is 3,600 times faster than a normal computer
- It uses qubits to perform calculations and solve optimisation problems
- In the video, Google and Nasa explain the basics of quantum computing
- They discuss multi-verse theory and give an example of optimisation
- Faster speeds mean it can tackle complex problems such as disease, climate change and genetics
- Google hopes it will help develop sophisticated artificial life, and find aliens

Daily Mail, 15 October 2013

This talk

1. A brief introduction to the quantum computing model
2. Quantum algorithms: what quantum computers can do
3. Experimental implementations
4. Further reading

The quantum model: qubits

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Pic: coins-of-the-uk.co.uk

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- ▶ A **qubit** (“quantum bit”) is stored in a tiny physical system like an individual atom that behaves **quantum mechanically**.



The quantum model: qubits

As well as being in states corresponding to 0 or 1, a qubit can be anywhere in between!

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This allows a quantum computer to run an algorithm on many possible inputs **simultaneously**.

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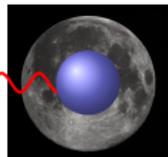
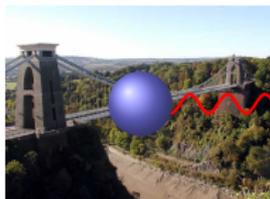
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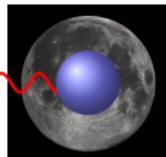
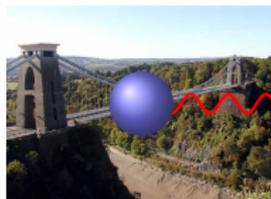


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- ▶ But what if the first qubit is in Bristol, and the second is on the Moon?



- ▶ It seems that the measurement result in Bristol has **instantaneously affected** the qubit on the Moon. . .
- ▶ This bizarre phenomenon is known as **quantum entanglement**.

Shor's algorithm

Integer factorisation

Given an integer N such that $N = p \times q$ for prime numbers p and q , find p and q .

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Shor's algorithm

- ▶ In 1994, Peter Shor described a **quantum** algorithm which can factorise large integers efficiently.
- ▶ No efficient classical algorithm is known for this problem.



Pic: physik.uni-graz.at

Factorisation and cryptography

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- ▶ The **RSA cryptosystem** which underlies Internet security relies on the hardness of integer factorisation.

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- ▶ If we could factorise large numbers efficiently, we could break this cryptosystem.

Factorisation and cryptography

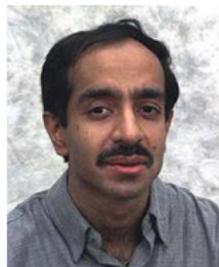
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In 2009, a 232-digit number was factorised using hundreds of computers over a period of **2 years**. . . by comparison, a large quantum computer could factorise a number with thousands of digits in a matter of **minutes**.

Grover's algorithm

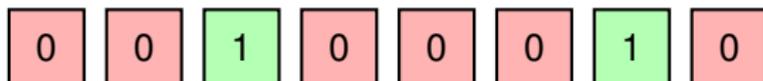
- ▶ One of the most basic problems in computer science is **unstructured search**.



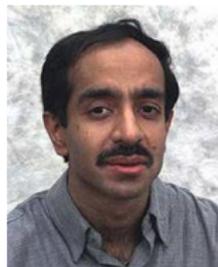
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Grover's algorithm

- ▶ One of the most basic problems in computer science is **unstructured search**.
- ▶ Imagine we have n boxes, each containing a 0 or a 1. We can look inside a box at a cost of one **query**.



- ▶ We want to find a box containing a 1.



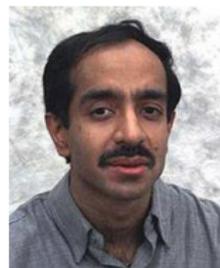
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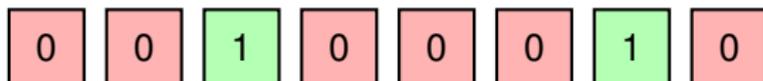
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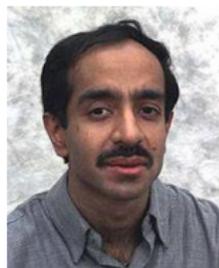
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Grover's algorithm

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- ▶ We want to find a box containing a 1.
- ▶ On a classical computer, this task could require n queries in the worst case. But on a quantum computer, **Grover's algorithm** can solve the problem with roughly \sqrt{n} queries.



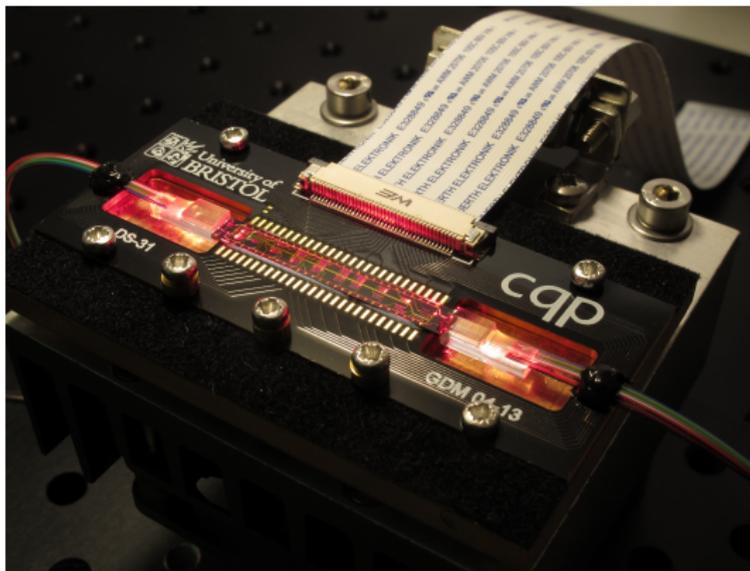
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Experimental implementations

There are a number of different technologies which could be used to implement a quantum computer.

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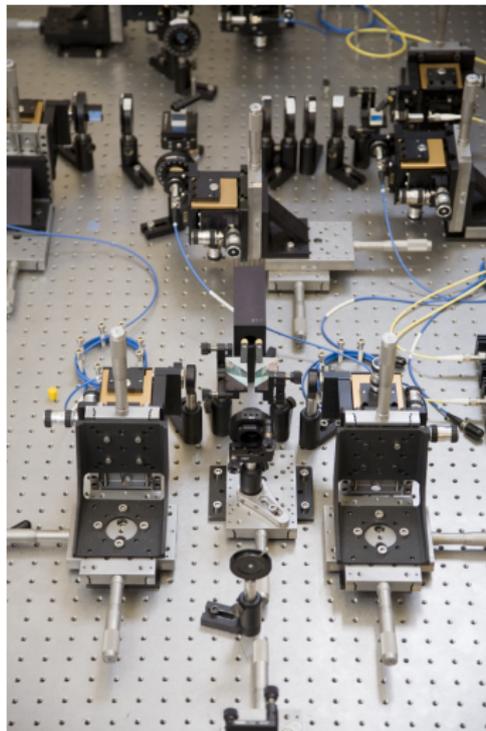
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Photonic quantum circuits on silicon (University of Bristol)

Pic: University of Bristol

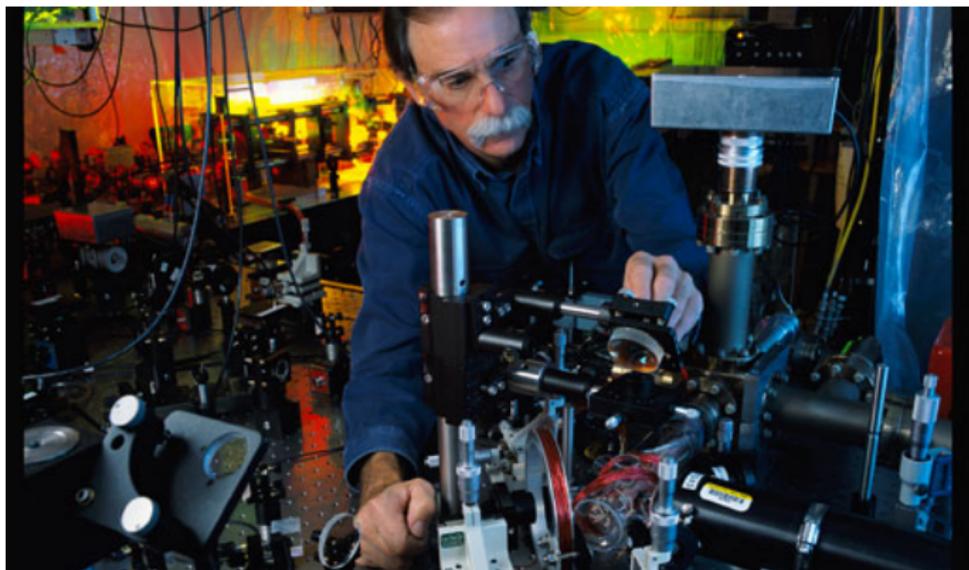
Experimental implementations



“Bulk” optics (University of Bristol)

Pic: Carmel King

Experimental implementations



Ion trap (David Wineland group, NIST)

Pic: nobelprize.org

Quantum computing FAQs

1. When can I have one?

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2. Will I have one on my desk?

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To summarise:

- ▶ Quantum computing is a new and exciting model of computation which can do things that classical computing simply cannot.
- ▶ A massive international effort is ongoing to build a large-scale quantum computer, including [here at Bristol](#).
- ▶ There are still many fascinating [open problems](#) to address.

Further reading

- ▶ **Winning a Game Show with a Quantum Computer**

Ashley Montanaro

<http://www.cs.bris.ac.uk/~montanar/gameshow.pdf>

- ▶ **Quantum Computing Since Democritus**

Scott Aaronson

<http://www.scottaaronson.com/democritus/>

- ▶ **Introduction to Quantum Computing, University of Waterloo**

John Watrous

<https://cs.uwaterloo.ca/~watrous/LectureNotes.html>

Partial timeline: Theory of quantum computing

- ⋮
- 1984 Quantum cryptographic key distribution invented [Bennett+Brassard]
- 1985 General quantum computational model proposed [Deutsch]
- 1992 First exponential quantum speed-up discovered [Deutsch and Jozsa]
- 1993 Quantum teleportation invented [Bennett et al.]
- 1994 Shor's algorithm rewrites the rulebook of classical cryptography
- 1995 Quantum error-correcting codes invented [Shor]
- 1996 Quantum simulation algorithm proposed [Lloyd]
- 1996 Quantum speed-up for unstructured search problems [Grover]
- 1998 Efficient quantum communication protocols [Buhrman et al.]
- 2003 Exponential speed-ups by quantum walks invented [Childs et al.]
- ⋮

Partial timeline: Quantum computing experiments

- ⋮
- 1997-8 Quantum teleportation demonstrated [Innsbruck, Rome, Caltech, ...]
- 1998 Quantum error-correction demonstrated [MIT]
- 2001 Shor's algorithm factorises $15 = 3 \times 5$ using NMR [IBM]
- 2005 8 qubits controlled in ion trap [Innsbruck]
- 2008 Photonic waveguide quantum circuits demonstrated [Bristol]
- 2010 Entangled states of 14 qubits created in ion trap [Innsbruck]
- 2012 $21 = 3 \times 7$ factorised using quantum optics [Bristol]
- 2012 $100\mu\text{s}$ coherence for superconducting electronic qubits [IBM]
- 2013 First publicly-accessible "quantum cloud" [Bristol]
- ⋮